# CS118 Exercises
# Prequel

**For this assignment, type all answers in the space provided – showing your work!**

1. The summation operator is the Greek letter sigma: Σ The common form for this operator is:

$$\sum_{i=j}^{n} f(i)$$

which is read as "Sum the computed values of *f(i)* for *i* starting at value *j* and continuing through value *n*". For example:

$$\sum_{i=3}^{6} (i+3)^2 = (3+3)^2 + (4+3)^2 + (5+3)^2 + (6+3)^2 = 36+49+64+81 = 230$$

Compute the following (show your work!)

a. $\sum_{i=1}^{8} i^2 - 3i + 2$

b. $\sum_{k=3}^{n} k^3$ (Show the indefinite calculation)

c. Infinite series are mathematical descriptions of how to compute irrational numbers such as π.
The computation isn't run forever; instead, it is either run to a certain precision or a certain number of terms are used.

Showing your work, compute the value of π using 10 terms from the series $\pi = \sqrt{12} \sum_{k=0}^{\infty} \frac{\left(\frac{-1}{3}\right)^k}{2k+1}$

2. A weighted average is an average where the individual components carry different significance. For example, when computing grades, individual *exam* grades are generally more important than individual *homework* grades. The formula is:

$$wa = \frac{\sum_i w_i v_i}{\sum_i w_i} \ni \begin{array}{l} w_i = \text{individual weight of item } i \\ v_i = \text{normalized value of item } i \end{array}$$

A more convenient formula when there are categories of items with weights for the categories:

$$wa = \frac{\sum_j w_j avg_j}{\sum_j w_j} \ni \begin{array}{c} w_j = \text{weight of category } j \\ avg_j = \text{normalized average of category } j \end{array}$$

In this version, the sum of the weights is typically 100 (although it doesn't have to be). For example: category 1 gets a weight of 20, category 2 gets a weight of 50 and category 3 gets a weight of 30. In that case the computation is:

$$wa = \frac{20\,avg_1 + 50\,avg_2 + 30\,avg_3}{20+50+30} = \frac{20\,avg_1 + 50\,avg_2 + 30\,avg_3}{100} = 0.2\,avg_1 + 0.5\,avg_2 + 0.3\,avg_3$$

It is important that all values are *normalized*, meaning they are values in the same possible range. Thus, if one score is out of 20 possible points and another is out of 30 possible points, they need to be normalized to a common range – typically this would mean the scores would be adjusted to be out of 100 – i.e. a percentage.

Compute the final grade where:

There are four exams and each exam is 15% of the final grade
There are 5 homework assignments with the average score counting as 20% of the final grade
There are 5 quizzes with each counting 2% of the final grade
There is a final project counting 10% of the final grade

| | |
|---|---|
| Scores: | |
| Exams: | 90, 80, 85, 94 |
| HW (each with different possible points): | 2 out of 3; 5 out of 7; 8 out of 10; 3 out of 4; 9 out of 20 |
| Quizzes (each out of 30): | 30, 25, 28, 14, 29 |
| Final project: | 88 |

3. Express the following concepts mathematically:

a) x is positive

b) z is non-negative

4. It's important to read and use wording very carefully. For example, if a percentage value changes from 12.5% to 10%, there has NOT been a change of 2.5%, or even -2.5%. We would say there has been "a change of 2.5 percentage points", or "decrease of 2.5 percentage points" or "a -2.5 percentage point change".

A "difference" implies a subtraction, as does a "change". "Difference" by itself does not imply a reference source, though, so it's important to determine if there is a base value from which the computation originates. For example, if a value changes **from** 12.5 **to** 10.0, we would say there has been "a 2.5 unit decrease" or that the change was "-2.5 units". In this case, we can tell that the reference (or base) value is 12.5. If we simply say "What's the difference between 12.5 and 10.0?" we aren't clearly specifying the base value, so the correct answer would be "2.5 units" and not "-2.5 units".

If we are talking about percentage it's important to understand the meaning of the words provided. There is much variation in how the terms "percent(age) difference", "percent change", "relative change" and "relative difference" are used. You must be clear on what is being requested and what information is known. For example, if we simply ask for the "percent difference between 10 and 15" the reference value is not clear. But if instead we ask for the "percent difference of 15 **from** 10", then it's clear that the reference (or base) value is 10 and we are requesting the amount of change as a percentage of the reference value.

Many students have gotten in the habit of looking up definitions on the Web. Not all definitions are correct or applied in the same manner. It is *very* important that you understand terms *before* they are required (like on an exam).

After all of that, the questions to answer (don't forget units):

a) What is the percentage difference of 80 from 30?

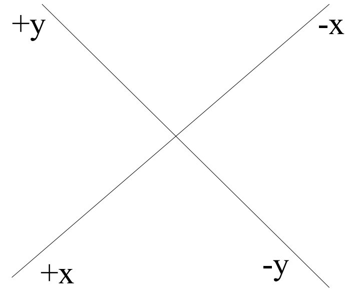
b) The temperature was 98°F and is now 75°F. What is the difference?

c) The temperature was 98°F and is now 75°F. What was the change?

d) The temperature was 98°F and is now 75°F. What was the relative change?

e) The temperature was 98°F and is now 75°F. What was the percentage change?

5. If we have a right-hand coordinate system with x and y axes like this:




In what direction is the +z axis?

6. A logarithm is an exponent. To compute the logarithm of a value means to find the exponent for some given base that when applied to the base will be the value. In other words:

If

(a) $\log_b V = x$

then

(b) $b^x = V$

Common bases are 10, 2, and *e* (the base of the natural logarithm). When written `log V`, the base is usually understood to be 10; $\log_e$ is frequently written as `ln`.

Compute the following values then use the result in an equation similar to form `(b)` above.

a. `log 100`

b. `log 64`

c. `ln 100`

d. `ln 64`

e. $\log_2 100$

f. $\log_2 64$

7. A line drawn in a normal Cartesian graph can be expressed as a relationship between an $x$-value and its corresponding $y$-value:

$$y = mx + b$$

where

$x$ is the independent variable
$y$ is the dependent variable
$m$ is the slope of the line
$b$ is the y-intercept of the line (the value where the line hits the y-axis – i.e. the value for $y$ when $x$ is 0)

The above formula is called the "slope-intercept" formula for the line.

The *slope* of a line is the ratio between the change in y values and the change in x value – in other words, for each unit of x change, how much does y change? The formula for computing this is:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

A line can also be expressed in the "point-slope" formula:

$$y - y_1 = m(x - x_1)$$

or

$$y = m(x - x_1) + y_1$$

where

$x$ is the independent variable
$y$ is the dependent variable
$x_1$ is a known (constant) value for the independent variable
$y_1$ is a known (constant) value for the dependent variable
$m$ is the slope of the line

a. Compute the slope for the lines passing through these data points: (1, 3) and (-5, 6)

b. Find the y-intercept for the line from part a.

c. Determine another two data points which lie on the line from part a.

d. Use one of your new data points and one of the original data points to compute the slope – it should of course be the same slope as part a.

e. Given these data points, compute the slope-intercept form of the line which passes through them: (1, 2) and (-5, 17)

f. Given these data points, compute the point-slope form of the line which passes through them: (2, 1) and (17, 5)

g. A line which intersects a second line at right-angles is said to be *perpendicular* or *orthogonal* to second line. The slopes of the two lines are related as:

$$m_1 m_2 = -1$$

Find two data points which lie on a line perpendicular to the line expressed in part f.